# SHENTON COLLEGE

# MATHEMATICS SPECIALIST UNIT 3 + 4

**Section Two**
**(Calculator-assumed)**

Your name _____

## Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

## Material required/recommended for this section

**To be provided by the supervisor**
Question/answer booklet for Section Two.
Formula Sheet (retained from Section One)

**To be provided by the candidate**
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in the WACE examinations

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
**before** reading any further.

## Structure of this examination

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---------|-------------------------------|-----------------------------------|------------------------|-----------------|--------------------|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| | | | **Total** | 150 | 100 |

## Instructions to candidates

The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2016.* Sitting this examination implies that you agree to abide by these rules

Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
  Fill in the number of the question(s) that you are continuing to answer at the top of the page.

**Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil, except in diagrams**.

## STRUCTURE OF THIS PAPER

| QUESTION | MARKS AVAILABLE | MARKS AWARDED |
|----------|-----------------|---------------|
| 9 | 6 | |
| 10 | 8 | |
| 11 | 7 | |
| 12 | 13 | |
| 13 | 10 | |
| 14 | 7 | |
| 15 | 8 | |
| 16 | 8 | |
| 17 | 7 | |
| 18 | 7 | |
| 19 | 10 | |
| 20 | 7 | |
| TOTAL | 98 | |

**Section Two: Calculator-assumed** **65% (97 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

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**Question 9** **(6 marks)**

A system of equations is shown below.

$$x + 2y + 3z = 1$$
$$y + 3z = -1$$
$$-y + (a^2 - 4)z = a + 2$$

(a) Determine the unique solution to the system when $a = 2$. (2 marks)

(b) Determine the value(s) of $a$ so that the system

(i) has an infinite number of solutions. (3 marks)

(ii) has no solutions. (1 mark)

**See next page**

**Question 10** **(8 marks)**

The length of time, $T$ months, that an athlete stays in an elite squad can be modelled by a normal distribution with population mean $\mu$ and population variance $\sigma^2 = 15$.

(a) An independent sample of five values of $T$ is $7.7, 15.2, 3.9, 13.4$ and $11.8$ months.

   (i) Calculate the mean of this sample and state the distribution that a large number of such samples is expected to follow. (2 marks)

   (ii) Use this sample to construct a 90% confidence interval for $\mu$, giving the bounds of the interval to two decimal places. (3 marks)

(b) Determine the smallest number of values of $T$ that would be required in a sample for the total width of a 95% confidence interval for $\mu$ to be less than 3 months. (3 marks)

**Question 11** **(7 marks)**

Plane $p_1$ has equation $3x + y + z = 6$ and line $l$ has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} - \mathbf{k})$.

(a) Show that the line $l$ lies in the plane $p_1$. (3 marks)

(b) Another plane, $p_2$, is perpendicular to plane $p_1$, parallel to the line $l$ and contains the point with position vector $i - 3j - k$. Determine the equation of plane $p_2$, giving your answer in the form $ax + by + cz = d$. (4 marks)

**Question 12** **(13 marks)**

(a) Show that the gradient of the curve $2x^2 + y^2 = 3xy$ at the point $(1, 2)$ is 2. (3 marks)

(b) Another curve passing through the point $(-2, 10)$ has gradient given by $\frac{dy}{dx} = \frac{2xy}{1+x^2}$.
Use a method involving separation of variables and integration to determine the equation of the curve. (4 marks)

**See next page**

(c) A particle is moving along the curve given by $y = \sqrt[3]{x}$, with one unit on both axes equal to one centimetre. When $x = 1$, the $y$-coordinate of the position of the particle is increasing at the rate of 2 centimetres per second.

    (i) Show that the $x$-coordinate is increasing at 6 centimetres per second at this instant.

                                                        (2 marks)

    (ii) Determine the exact rate at which the distance of the particle from the origin is changing at this instant.

                                                        (4 marks)
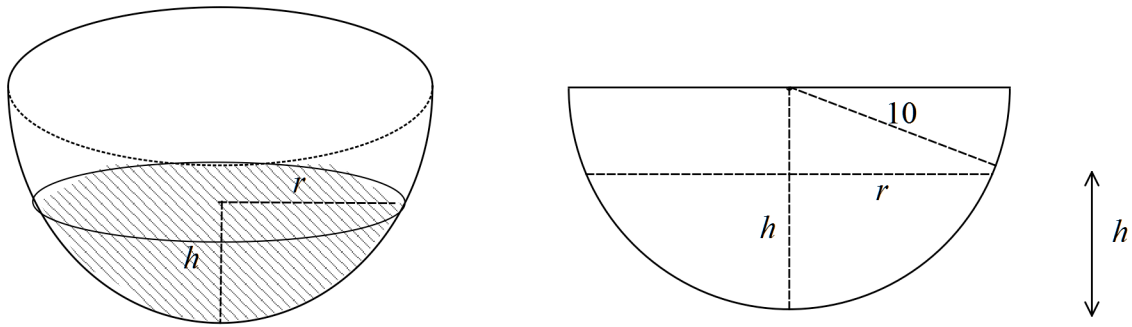
**See next page**

**Question 13** **(10 marks)**

(a)   The circle with equation $x^2 + (y-10)^2 = 100$ is shown in the diagram below.   (3 marks)

The shaded region of the circle is rotated about the y axis.
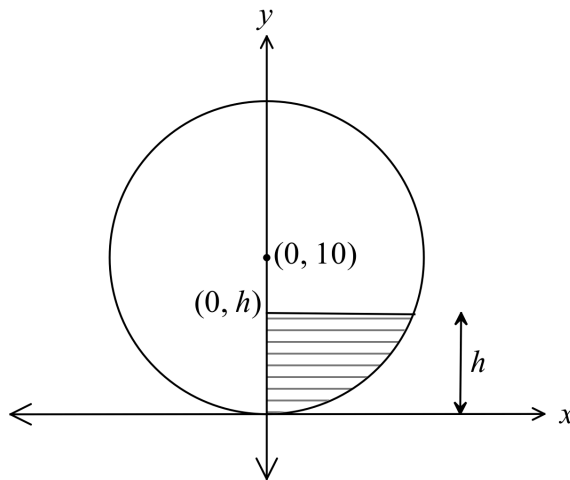


Show that the volume generated by the rotation is $V = \pi(10h^2 - \frac{h^3}{3})$.

(b)    A hemi-spherical vase of radius 10 cm is being filled with water at a rate of 10 cm$^3$ per second.



(i)    Find the relationship between $h$ and $r$.                    (1 mark)

(ii)   Use a calculus method to find the change in the radius $r$ if the height $h$ is increased from 8 cm to 8.1 cm.                    (3 marks)

(iii)  Find $\dfrac{dh}{dt}$ when $r = 8$ cm given $V = 10h^2 - \dfrac{h^3}{3}$.                    (3 marks)

**See next page**

**Question 14**       **(7 marks)**

(a)    The equation of a sphere with centre at $(2, -3, 1)$ is $x^2 + y^2 + z^2 = ax + by + cz - 2$.

Determine the values of $a, b, c$ and the radius of the circle.       (3 marks)

(b)    Two particles, P and Q, leave their initial positions at the same time and travel with constant velocities shown in the table below.

| Particle | Initial position | Velocity |
|----------|------------------|----------|
| P | $10\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$ | $6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ |
| Q | $28\mathbf{i} + 22\mathbf{j} - 31\mathbf{k}$ | $2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ |

Show that the two particles collide, stating the position vector of the point of collision.

(4 marks)

**Question 15** **(8 marks)**

(a) Briefly describe a reason that a sample rather than a complete population may be used when carrying out a statistical investigation. (1 mark)

(b) A researcher used government records to select a random sample of the ages of 114 men who had died recently in a town close to an industrial complex. The mean and standard deviation of the ages in the sample were 73.3 and 8.27 years respectively.

   (i) Explain why the sample standard deviation is a reasonable estimate for the population standard deviation in this case. (1 mark)

   (ii) Calculate a 98% confidence interval for the population mean and explain what the interval shows. (4 marks)

   (iii) The national average life-span of men was known to be 75.3 years. State with a reason what conclusion the researcher could draw from the confidence interval calculated in (ii) about the life-span of men in the town. (2 marks)

**Question 16** **(8 marks)**

(a)    On the Argand diagram below, clearly show the region that satisfies the complex inequalities given by $|z + 3 - 4i| \leq 5$ and $\frac{\pi}{2} \leq \arg z \leq \pi$.    (4 marks)
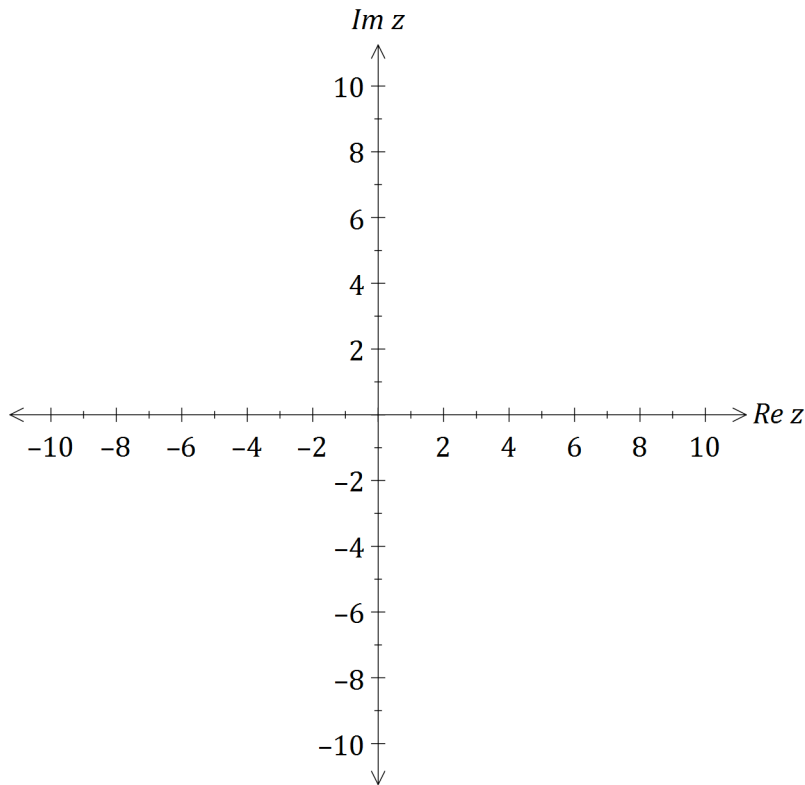


(b)    Determine all roots of the equation $z^5 = 16\sqrt{3} + 16i$, expressing them in the form $r \operatorname{cis} \theta$, where $r \geq 0$ and $-\pi \leq \theta \leq \pi$.    (4 marks)
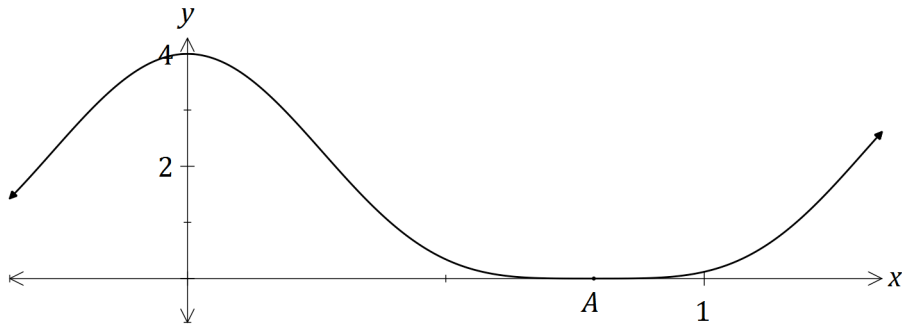
**Question 17** **(7 marks)**

(a) A market research company conducted a study on whether the packaging of breakfast cereals had an effect on the perceived healthiness of the product. Samples of 40 shoppers were shown three different boxes of cereal (which all had identical ingredients) and asked to rank them from 1 – 10 based on how healthy they thought they were. The first box, which was printed in colour and featured cartoon brand mascots, scored an average of 7.5. The second box, also in colour but showing scenes of a fruit orchard, scored 7.8. The third box, which was made of unbleached brown card and featuring simple icons representing wheat and nuts, scored 8.5. The standard deviations of all three samples was very close to 2.4.

    (i) Determine the 90% confidence intervals for each of the three boxes. (3 marks)

    (ii) Comment on the statistical significance of the effect of the packaging, based on your 90% confidence levels. (2 marks)

(b) Initial samples taken to determine the growth rates of farmed Murray cod measured 100 adult fish and found that the weights of the fish were normally distributed with a mean of 745g and a standard deviation of 50g.
A new sample is to be taken for which the investigators wish to be 95% confident that the mean weight of this second sample is within 7g of the population mean.
Assuming that the sample standard deviation is an accurate predictor of the population standard deviation, what should they use as the size of this second sample. (2 marks)

**Question 18**        **(7 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = 4\cos^4(2x)$ and $A$ is the smallest root of $f(x)$, $x > 0$.



(a)     Show that $4\cos^4(2x) = \dfrac{3 + 4\cos(4x) + \cos(8x)}{2}$.        (3 marks)

(b)     Hence determine $\int 4\cos^4(2x)\,dx$.        (2 marks)

(c)     Determine the exact volume of the solid generated when the region bounded by $y = f(x)$, $y = 0$, $x = 0$ and $x = A$ is rotated through 360° about the $x$-axis.        (2 marks)

**See next page**

**Question 19** **(10 marks)**

(a) A small object has initial position vector $\mathbf{r}(0) = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$ metres and moves with velocity vector given by $\mathbf{v}(t) = 2t\mathbf{i} - 4t\mathbf{j} + 3\mathbf{k}$ ms$^{-1}$, where $t$ is the time in seconds.

   (i) Show that the acceleration of the object is constant and state the magnitude of the acceleration. (2 marks)

   (ii) Determine the position vector of the object after 2 seconds. (3 marks)

(b) Another small object has position vector given by $\mathbf{r}(t) = (1 + 2\sec t)\mathbf{i} + (3\tan t - 2)\mathbf{j}$ m, where $t$ is the time in seconds.

(i) Determine the distance of the object from the origin when $t = \frac{\pi}{3}$.      (2 marks)

(ii) Derive the Cartesian equation of the path of this object.      (3 marks)

**Question 20** (7 marks)

(a) A particle undergoing simple harmonic motion with a period of 5 seconds is observed to move in a straight line, oscillating 3.6 m either side of a central position. Determine the speed of the particle when it is 3 m from the central position. (3 marks)

(b) Another particle moving in a straight line experiences an acceleration of $x + 2.5$ ms$^{-2}$, where $x$ is the position of the particle at time $t$ seconds.

Given that when $x = 1$, the particle had a velocity of 2 ms$^{-1}$, determine the velocity of the particle when $x = 2$. (4 marks)

**End of questions**

**Additional working space**

Question number: _____

**Additional working space**

Question number: _____